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Introduction to Upside-Down Logic: Its Deep Relation to Neutrosophic Logic and Applications

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Abstract

In the study of uncertainty, concepts such as fuzzy sets [113], fuzzy graphs [79], and neutrosophic sets [88] have been extensively investigated. This paper focuses on a novel logical framework known as Upside-Down Logic, which systematically transforms truths into falsehoods and vice versa by altering contexts, meanings, or perspectives. The concept was first introduced by F. Smarandache in [99].

To contribute to the growing interest in this area, this paper presents a mathematical definition of Upside-Down Logic, supported by illustrative examples, including applications related to the Japanese language. Additionally, it introduces and explores Contextual Upside-Down Logic, an advanced extension that incorporates a contextual transformation function, enabling the adjustment of logical connectives in conjunction with flipping truth values based on contextual shifts. Furthermore, the paper introduces Indeterm-Upside-Down Logic and Certain Upside-Down Logic, both of which expand Upside-Down Logic to better accommodate indeterminate values. Finally, a simple algorithm leveraging Upside-Down Logic is proposed and analyzed, providing insights into its computational characteristics and potential applications.

Keywords: Upside-Down Logic, Neutrosophic Logic, Logic, Fuzzy Logic, Japanese

1 Short Introduction

1.1 Uncertain Logic and Upside-Down Logic

Uncertainty in real-world events is modeled using mathematical concepts. In the field of logic (cf. [21, 103]), various frameworks have been developed to address uncertainty, including Fuzzy Logic [113–115], Neutrosophic Logic [88, 90, 94], and Plithogenic Logic [93, 102]. For example, Neutrosophic Logic extends classical logic by introducing three degrees—truth, indeterminacy, and falsity—accommodating uncertainty and contradictions simultaneously. These uncertain logics have also been extended to concepts such as sets [89, 100] and graphs [34, 37–39], resulting in numerous studies parallel to advancements in the logic domain.

This paper focuses on a logical framework called Upside-Down Logic, which systematically transforms truths into falsehoods and vice versa by altering contexts, meanings, or perspectives. This logical concept was introduced by F. Smarandache in [99]. A central focus is the phenomenon of reversals caused by ambiguity. In decision-making, individuals strive to discern what is correct or incorrect. However, ambiguity can lead to situations where something initially perceived as correct ultimately proves to be incorrect, causing misunderstandings or unfortunate outcomes. Simply put, Upside-Down Logic formalizes this phenomenon into a structured logical framework.

1.2 Contributions of This Paper

This subsection explains the contributions of this paper. As discussed above, research on Upside-Down Logic is significant but still in its early stages. This paper aims to advance this emerging field by presenting a mathematical definition of Upside-Down Logic, accompanied by several illustrative examples, including applications related to the Japanese language.

Additionally, as a related concept, this paper introduces and explores Contextual Upside-Down Logic. Contextual Upside-Down Logic extends Upside-Down Logic by incorporating a contextual transformation function that not only flips truth values but also adjusts logical connectives based on the given context.

Moreover, this paper introduces Indeterm-Upside-Down Logic and Certain Upside-Down Logic, both of which extend Upside-Down Logic to better handle indeterminate values. Additionally, a simple algorithm utilizing Upside-Down Logic is examined.

These contributions can be applied to various logical frameworks, such as Neutrosophic Logic, and have potential applications in decision-making and other related fields.

1.3 The Structure of the Paper

The structure of this paper is as follows. Section 2 introduces the concept of Upside-Down Logic. Section 3 delves into Contextual Upside-Down Logic. Section 4 discusses Indeterm-Upside-Down Logic. Section 5 focuses on Certain Upside-Down Logic. Finally, Section 6 outlines future directions for this research.

2 Upside-Down Logic

In this section, we explore the mathematical definition of Upside-Down Logic, real-life examples, basic theorems, its application to Neutrosophic Logic, and its associated algorithms.

2.1 Basic Definition of Formal Language

To explore Upside-Down Logic, several key concepts are introduced below. For further details, readers are encouraged to consult the respective lecture notes and surveys on these topics (ex. [33, 42, 43, 50, 62]).

Definition 2.1 (Set)**.** [50] A *set* is a collection of distinct objects, known as elements, that are clearly defined, allowing any object to be identified as either belonging to or not belonging to the set. If A is a set and x is an element of A, this membership is denoted by $x \in A$. Sets are typically represented using curly brackets.

Definition 2.2 (Formal Language)**.** [42, 78] A *formal language* $\mathcal{L}$ is defined as a set of strings (or sequences) formed from a finite alphabet Σ, subject to specific syntactic rules. Formally:

$$\mathcal{L} \subseteq \Sigma^*,$$

where Σ^* is the set of all finite strings over the alphabet Σ. The strings in $\mathcal{L}$ are called *well-formed formulas (WFFs).*

A formal language $\mathcal{L}$ is typically accompanied by:

- A set of *symbols* (or *alphabet*) Σ, which may include logical connectives (e.g., $\wedge$, $\vee$, $\neg$), quantifiers (e.g., $\forall$, $\exists$), variables, and parentheses.
- A set of *formation rules* that determine which strings in Σ^* are well-formed.

Example 2.3 (Japanese as a Formal Language)**.** Consider the Japanese language as a formal language $\mathcal{L}$. Note that Readers interested in further details on Japanese linguistics are encouraged to consult references such as [66, 67, 105].

Let:

- Σ be the alphabet, including:
  - Hiragana symbols, e.g., {あ, い, う, え, お, . . . },
  - Katakana symbols, e.g., {ア, イ, ウ, エ, オ, . . . },
  - Kanji characters, e.g., {日, 本, 語, . . . },
  - Romanized characters and punctuation, e.g., $\{a, b, c, \ldots, ., !, ?\}$.
- Σ^* be the set of all possible finite sequences of these symbols.
- Formation rules define grammatically correct sentences in Japanese. Examples include:
  - Subject-Object-Verb order, such as:

    私 (I) + りんご (apple) + を (object marker) + 食べます (eat).

  - Use of particles (e.g., は, が, を) to indicate grammatical roles:

    犬 (dog) + が (subject marker) + 走る (run).

  - Proper use of politeness levels, e.g., です or ます forms.

Thus, $\mathcal{L}$ represents all well-formed Japanese sentences that adhere to these grammatical rules.

Example 2.4 (Mathematical Identity as a Formal Language)**.** Consider the mathematical identity A: "2+2 = 4" as a proposition in a formal language $\mathcal{L}$, where:

- The alphabet Σ consists of symbols for numbers, operators, and equality, e.g., $\Sigma = \{0, 1, 2, 3, 4, +, =, \ldots\}$.
- The formation rules are derived from the axioms and rules of standard arithmetic over the real numbers.

Definition 2.5 (Logical System)**.** (cf. [58]) A *logical system* $\mathcal{M}$ is a mathematical structure that formalizes reasoning. It consists of:

$$\mathcal{M} = (\mathcal{P}, \mathcal{V}, v),$$

where:

- $\mathcal{P}$ is the set of propositions (or statements) in the formal language $\mathcal{L}$.
- $\mathcal{V}$ is the set of truth values, such as $\{\text{True}, \text{False}\}$ for classical logic.
- $v : \mathcal{P} \to \mathcal{V}$ is a *valuation function* (or interpretation function) that assigns a truth value to each proposition in $\mathcal{P}$.

In addition, a logical system may include:

- A set of *axioms* $\mathcal{A} \subseteq \mathcal{P}$ that are assumed to be true within the system.
- A set of *inference rules* $\mathcal{I}$ that define valid transformations of propositions to derive new truths.

2.2 Formal Definition of Upside-Down Logic

In this subsection, we examine the mathematical definition of Upside-Down Logic. The related definitions and concepts are outlined below.

Notation 2.6. *Let $\mathcal{P}$ be a set of propositions, and let C be a set of contexts. Let*

$$T : \mathcal{P} \times C \to \{\mathit{True}, \mathit{False}, \mathit{Indeterminate}\}$$

be a truth valuation function that assigns a truth value to each proposition-context pair.

Notation 2.7. *Let $\mathcal{L}$ be a formal language, and let $\mathcal{M}$ be a logical system with a set of propositions $\mathcal{P}$, a set of truth values $\mathcal{V}$, and a valuation function $v : \mathcal{P} \to \mathcal{V}$.*

Definition 2.8 (Upside-Down Logic)**.** [99] An *Upside-Down Logic* is a logical system $\mathcal{M}'$ derived from $\mathcal{M}$ by introducing a transformation U on propositions and/or contexts such that:

1. For any proposition $A \in \mathcal{P}$ with truth value $v(A)$ in context C, there exists a transformed proposition $U(A)$ and/or transformed context $U(C)$ where:
   - *Falsification of the Truth*: If $v(A) = \text{True}$ in C, then $v(U(A)) = \text{False}$ in $U(C)$.
   - *Truthification of the False*: If $v(A) = \text{False}$ in C, then $v(U(A)) = \text{True}$ in $U(C)$.
2. The transformation U is well-defined and consistent within the logical system $\mathcal{M}'$.

Definition 2.9 (Context)**.** [99] A *context* C is a set of parameters or conditions under which propositions are evaluated. This may include spatial, temporal, semantic, or interpretative settings.

Definition 2.10. The Upside-Down Transformation U may involve one or more of the following operations:

- *Changing the Domain*: Modifying the domain or universe of discourse.
- *Altering Attributes*: Changing properties or characteristics of elements within propositions.
- *Reversing Logical Operations*: Applying dual operators or complement functions.
- *Contextual Shifts*: Altering the context C to $U(C)$ such that the truth value of A changes.
- *Semantic Reinterpretation*: Reinterpreting the meanings of terms or predicates.

2.3 Some Example of Upside-Down Logic

In this subsection, we explore some examples of Upside-Down Logic, such as Mathematical Identity and Physical Laws.

Example 2.11 (Mathematical Identity)**.** Let A: "$2 + 2 = 4$" in the context C of standard arithmetic over the real numbers. In context C:

$$2 + 2 = 4 \implies v(A, C) = \text{True}.$$

Falsification of the Truth: Define a new context $U(C)$ where arithmetic is modulo 3.

In context $U(C)$:

$$2 + 2 = 1 \pmod 3 \implies v(A, U(C)) = \text{False}.$$

Alternatively, consider a proposition A: "$2 + 2 = 0$" in context C where arithmetic is modulo 4.

In context C:

$$2 + 2 = 0 \pmod 4 \implies v(A, C) = \text{True}.$$

Truthification of the False: In standard arithmetic context $U(C)$, where arithmetic is over the integers, $2+2 = 4$, so $v(A, U(C)) = \text{False}$.

Example 2.12 (Physical Laws)**.** Let A: "Water boils at 100°C." In context C: "At standard atmospheric pressure (1 atm)."

$$\text{Boiling point of water at 1 atm is } 100^\circ\text{C} \implies v(A, C) = \text{True}.$$

Falsification of the Truth: Change the context to $U(C)$: "At an altitude of 3,000 meters where atmospheric pressure is approximately 0.7 atm."

At this pressure, water boils at approximately 90°C.

$$v(A, U(C)) = \text{False}.$$

Truthification of the False: Let B: "Water boils at 90°C." In context C: "At sea level (1 atm)."

$$\text{Boiling point is } 100^\circ\text{C} \implies v(B, C) = \text{False}.$$

In transformed context $U(C)$: "At an altitude of 3,000 meters (0.7 atm)."

$$\text{Boiling point is } 90^\circ\text{C} \implies v(B, U(C)) = \text{True}.$$

Example 2.13 (Linguistic Ambiguity)**.** Let A: "The bank is open today." In context C: "Referring to a financial institution on a weekday."

$$\text{If today is a weekday, } v(A, C) = \text{True}.$$

Falsification of the Truth: Change the context to $U(C)$: "Referring to the river bank in a natural reserve area that is closed to the public."

$$v(A, U(C)) = \text{False}.$$

Semantic Reinterpretation: The word "bank" shifts meaning from "financial institution" to "river bank," changing the truth value.

Example 2.14 (Cultural Context)**.** Let A: "Eating beef is acceptable." In context C: "In a Western country where eating beef is a common practice."

$$v(A, C) = \text{True}.$$

Falsification of the Truth: Change the context to $U(C)$: "In India where cows are considered sacred by Hindus."

$$v(A, U(C)) = \text{False}.$$

2.4 Example of Upside-Down Logic on Japanese Culture

In this subsection, we examine examples of Upside-Down Logic within the context of the Japanese language. Known for its high degree of ambiguity, Japanese linguistics have been the subject of extensive research on resolving linguistic ambiguities (cf. [24,70,71,107]). Several illustrative examples are provided below. Readers interested in further details on Japanese linguistics are encouraged to consult references such as [66, 67, 105].

Example 2.15 (Ambiguity in Japanese Expressions)**.** In Japanese, the phrase "*sore wa chotto...*" literally means "that is a little...," trailing off without completing the sentence [65].

Let A: "The speaker is refusing the request."

In context C: The literal interpretation suggests hesitation without explicit refusal:

$$v(A, C) = \text{False}.$$

Context Transformation: Understanding cultural nuances in context $U(C)$, the phrase is recognized as a polite way to decline:

$$v(A, U(C)) = \text{True}.$$

Upside-Down Logic Transformation: Using Upside-Down Logic, the truth value of A flips when shifting from C to $U(C)$.

What appears as hesitation is actually a polite refusal in the transformed context.

Example 2.16 (Proverbs and Contradictions in Japanese)**.** Japanese proverb: *Makeru ga kachi* —"Defeat is victory." [1]

Let A: "Defeat leads to victory."

In context C: Literally interpreted, the proposition appears contradictory:

$$v(A, C) = \text{False}.$$

Context Transformation: In the cultural context $U(C)$, the proverb is understood to mean that yielding can lead to ultimate success:

$$v(A, U(C)) = \text{True}.$$

Upside-Down Logic Transformation: The truth value of A flips when moving from the literal context to the cultural context.

The conventional understanding of winning and losing is inverted through the proverb's cultural meaning.

[1] Japanese proverbs are traditional sayings reflecting cultural values, wisdom, and life lessons, often using metaphorical or poetic expressions [16].

Example 2.17 (Honne and Tatemae)**.** In Japanese culture, *Honne* refers to a person's true feelings, while *Tatemae* refers to the public façade (cf. [104, 111]).

Let A: "He agrees with the proposal."

In context C: Based on his public behavior (*Tatemae*), the proposition appears to be true:

$$v(A, C) = \text{True}.$$

Context Transformation: When shifting to the context of *Honne* (private feelings) $U(C)$, he may actually disagree:

$$v(A, U(C)) = \text{False}.$$

Upside-Down Logic Transformation: Using Upside-Down Logic, we can define the transformed proposition:

$$U(A) : \text{"He disagrees with the proposal internally."}$$

In the context $U(C)$:

$$v(U(A), U(C)) = \text{True}.$$

Agreement in public (*Tatemae*) is inverted to disagreement in private feelings (*Honne*) under the transformed context.

Example 2.18 (Omote and Ura (Front and Back))**.** In Japanese aesthetics, *Omote* means the front or public face, while *Ura* refers to the back or hidden side (cf. [49, 51]).

Let A: "The painting is simple."

In context C: Observing the *Omote* (front) of the painting, the proposition holds true:

$$v(A, C) = \text{True}.$$

Context Transformation: Exploring the *Ura* (hidden meanings) in context $U(C)$, new complexities are revealed:

$$v(A, U(C)) = \text{False}.$$

Upside-Down Logic Transformation: We can redefine the proposition to reflect the depth:

$$U(A) : \text{"The painting is complex with deep symbolism."}$$

In the transformed context $U(C)$:

$$v(U(A), U(C)) = \text{True}.$$

The simplicity observed in the *Omote* is inverted into complexity when considering the *Ura*.

Example 2.19 (The Concept of *Ma*)**.** *Ma* refers to the space or pause between objects or moments, an essential concept in Japanese aesthetics.

Let A: "Silence indicates agreement."

In context C: In some cultures, silence may be interpreted as agreement:

$$v(A, C) = \text{True}.$$

Context Transformation: In the Japanese context $U(C)$, silence can signify various meanings, including disagreement or contemplation:

$$v(A, U(C)) = \text{False}.$$

Upside-Down Logic Transformation: Reformulating the proposition to reflect this ambiguity:

$$U(A) : \text{"Silence does not necessarily indicate agreement."}$$

In the context $U(C)$:

$$v(U(A), U(C)) = \text{True}.$$

The interpretation of silence as agreement is inverted in the Japanese context, highlighting the cultural nuances of *Ma*.

Example 2.20 (The Art of Haiku)**.** Haiku is a form of Japanese poetry that captures a moment with brevity and depth (cf. [80, 109]).

Let A: "Short poems lack depth."

In context C: In the general context of poetry, depth is often associated with longer and more complex forms. Thus:

$$v(A, C) = \text{True}.$$

Context Transformation: Consider the context of Japanese Haiku, $U(C)$, where brevity is seen as a means to achieve profound depth. In this new context, the original proposition A no longer holds true:

$$v(A, U(C)) = \text{False}.$$

Upside-Down Logic Transformation: Using Upside-Down Logic, we transform the proposition to:

$$U(A) : \text{"Short poems convey profound depth."}$$

Under the transformed context $U(C)$, this new proposition aligns with the values of Haiku and becomes true:

$$v(U(A), U(C)) = \text{True}.$$

2.5 Some Basic Theorem of Upside-Down Logic

In this subsection, we consider some basic theorems of Upside-Down Logic. The following theorems hold.

Theorem 2.21 (Invariance under Fixed Context)**.** *Within a fixed context C, the Upside-Down Transformation U must alter the proposition A to change its truth value.*

Proof. Assume that the context C is fixed. To change the truth value of A, the transformation U must alter A to $U(A)$ such that $v(U(A), C) \neq v(A, C)$.

If U does not alter A, then $v(A, C) = v(U(A), C)$, and the truth value remains the same.

Therefore, to achieve Falsification or Truthification within a fixed context, the proposition itself must be altered. □

Theorem 2.22 (Composition of Upside-Down Transformations)**.** *The composition of two Upside-Down Transformations may not necessarily return to the original truth value.*

Proof. Let U_1 and U_2 be two different Upside-Down Transformations.

Applying U_1 and then U_2:

$$v(U_2(U_1(A)), U_2(U_1(C))) = v^{(2)}(A).$$

Since each transformation may alter the context and/or the proposition differently, the final truth value may not be equal to the original $v(A, C)$.

Therefore, in general:

$$v(U_2(U_1(A)), U_2(U_1(C))) \neq v(A, C).$$

□

Theorem 2.23 (Non-Idempotence of Upside-Down Transformations)**.** *An Upside-Down Transformation U is not necessarily idempotent, i.e., applying U twice does not guarantee that the original proposition and context are restored.*

Proof. Consider U that alters the context from C to $U(C)$. Applying U again:

$$U(U(C)) = U^2(C).$$

Unless U is specifically defined such that $U^2(C) = C$ and $U^2(A) = A$, the original proposition and context are not restored.

Therefore, in general, U is not idempotent. □

Corollary 2.24. *If U is involutive (i.e., $U^2 = id$), then applying U twice returns to the original proposition and context.*

Proof. If U is such that $U(U(A)) = A$ and $U(U(C)) = C$, then:

$$v(U(U(A)), U(U(C))) = v(A, C).$$

Thus, the original truth value is restored. □

2.6 Relationship between Neutrosophic Logic and Upside-Down Logic

In this subsection, we explore the relationship between Neutrosophic Logic and Upside-Down Logic. First, we present the definition of Neutrosophic Logic below [36, 88]. Note that Neutrosophic Logic is known to generalize Fuzzy Logic (cf. [88]).

Definition 2.25 (Neutrosophic Logic)**.** [88] Neutrosophic Logic extends classical logic by assigning to each proposition a truth value comprising three components:

$$v(A) = (T, I, F),$$

where $T, I, F \in [0, 1]$ represent the degrees of truth, indeterminacy, and falsity, respectively.

Neutrosophic Logic and related concepts (e.g., Neutrosophic Set) are known for their ability to model a wide range of phenomena [6, 25, 27, 54, 64, 74, 106, 110, 112, 116, 117]. As a reference, several examples of applying Neutrosophic Logic in real-life contexts are provided below.

Example 2.26 (Medical Diagnosis)**.** In medical diagnosis, Neutrosophic Logic is used to model uncertain and incomplete data. For example, a proposition like ”The patient has Disease X” can be evaluated as $v(A) = (0.6, 0.2, 0.2)$, where:

- $T = 0.6$: There is a 60% likelihood the patient has the disease.
- $I = 0.2$: 20% of the data is inconclusive.
- $F = 0.2$: There is a 20% likelihood the patient does not have the disease.

This helps in combining test results, symptoms, and expert opinions to make better-informed decisions(cf. [13, 19, 26]).

Example 2.27 (Decision-Making in Business)**.** In business decision-making, Neutrosophic Logic is used to evaluate competing strategies under uncertain conditions. For instance, when deciding whether to invest in a project, a proposition like ”The project will yield profit” might have a value $v(A) = (0.7, 0.1, 0.2)$, indicating:

- $T = 0.7$: A 70% chance the project will be profitable.
- $I = 0.1$: A 10% level of uncertainty due to incomplete market data.
- $F = 0.2$: A 20% chance the project will not be profitable.

This allows decision-makers to weigh risks and rewards more effectively (cf. [2, 84]).

Example 2.28 (Artificial Intelligence and Robotics)**.** In AI and robotics, Neutrosophic Logic is used to model complex reasoning in uncertain environments. For instance, a robot navigating a dynamic environment can evaluate a proposition like ”The path ahead is clear” as $v(A) = (0.8, 0.1, 0.1)$:

- $T = 0.8$: 80% confidence the path is clear.
- $I = 0.1$: 10% uncertainty due to sensor noise.
- $F = 0.1$: 10% likelihood the path is obstructed.

This helps in planning and adapting to changes in real-time(cf. [7]).

Example 2.29 (Social Network Analysis)**.** In social network analysis, Neutrosophic Logic is applied to measure trustworthiness in online interactions. For a proposition like ”User X is trustworthy,” a truth value $v(A) = (0.5, 0.3, 0.2)$ might represent:

- $T = 0.5$: A 50% degree of trust based on previous interactions.
- $I = 0.3$: 30% uncertainty due to a lack of sufficient data.
- $F = 0.2$: A 20% indication of untrustworthiness from contradictory feedback.

This helps in filtering content and detecting fraudulent activities (cf. [82, 83]).

The relationship between Neutrosophic Logic and Upside-Down Logic is outlined below. Readers should note that the application examples of Upside-Down Logic presented here are merely illustrative. It is hoped that further concrete investigations into their connection will be undertaken in the future.

Theorem 2.30 (Upside-Down Transformation in Neutrosophic Logic)**.** *In Neutrosophic Logic, the Upside-Down Transformation U corresponds to interchanging the truth and falsity components while possibly adjusting indeterminacy.*

Proof. Define U such that for any proposition A:

$$U(v(A)) = (F, I', T),$$

where I' is determined based on the specific transformation rules (e.g., $I' = I$ or $I' = 1 - I$). This transformation flips the degrees of truth and falsity, mirroring the Upside-Down Logic concept. □

Example 2.31 (Upside-Down Transformation in Neutrosophic Logic)**.** Let $v(A) = (0.8, 0.1, 0.1)$. Applying U:

$$U(v(A)) = (0.1, I', 0.8).$$

If we let $I' = I = 0.1$, then:

$$U(v(A)) = (0.1, 0.1, 0.8).$$

Thus, the proposition that was mostly true becomes mostly false under U.

Theorem 2.32 (Preservation of Indeterminacy)**.** *If the Upside-Down Transformation preserves the indeterminacy component, then U is an involutive transformation in Neutrosophic Logic.*

Proof. Assuming $I' = I$ and defining U as:

$$U(v(A)) = (F, I, T).$$

Applying U twice:

$$U(U(v(A))) = U(F, I, T) = (T, I, F) = v(A).$$

Therefore, U is involutive. □

Upside-Down Transformation can similarly be applied to concepts analogous to Neutrosophic Logic. As an example, we consider the application of Upside-Down Transformation to Double-Valued Neutrosophic Logic, as outlined below (cf. [52, 59]). Readers should note that the application examples of Upside-Down Logic presented here are merely illustrative.

Definition 2.33 (Double-Valued Neutrosophic Logic)**.** Let X be a space of points (or objects) where each $x \in X$ represents an element. A *Double-Valued Neutrosophic Set (DVNS) A* is characterized by:

$$A = \{(x, T_A(x), I_T(x), I_F(x), F_A(x)) : x \in X\},$$

where:

- $T_A(x) \in [0, 1]$ is the *truth membership value*,
- $I_T(x) \in [0, 1]$ is the *indeterminacy leaning towards truth*,
- $I_F(x) \in [0, 1]$ is the *indeterminacy leaning towards falsity*,
- $F_A(x) \in [0, 1]$ is the *falsity membership value*.

These values satisfy the condition:

$$0 \le T_A(x) + I_T(x) + I_F(x) + F_A(x) \le 4.$$

As a reference, several examples of utilizing Double-Valued Neutrosophic Logic in real-life contexts are provided below.

Example 2.34 (Medical Treatment Effectiveness Evaluation)**.** Consider a medical study evaluating the effectiveness of a new drug. For each patient x, the Double-Valued Neutrosophic Set A represents the assessment of the statement: "The drug is effective for the patient."

$$A = \{(x, T_A(x), I_T(x), I_F(x), F_A(x)) : x \in X\},$$

where:

- $T_A(x) = 0.8$: 80% evidence supports the drug's effectiveness.
- $I_T(x) = 0.1$: 10% uncertainty leans towards effectiveness due to inconclusive lab results.
- $I_F(x) = 0.05$: 5% uncertainty leans towards ineffectiveness due to side effects.
- $F_A(x) = 0.05$: 5% evidence indicates the drug is not effective.

This representation allows researchers to account for both supporting and opposing evidence, as well as uncertainty, in evaluating the drug's performance.

Example 2.35 (Customer Sentiment Analysis)**.** In e-commerce, Double-Valued Neutrosophic Logic can be used to analyze customer sentiments about a product. For each customer x, the Double-Valued Neutrosophic Set A evaluates the statement: "The customer is satisfied with the product."

$$A = \{(x, T_A(x), I_T(x), I_F(x), F_A(x)) : x \in X\},$$

where:

- $T_A(x) = 0.7$: 70% of the customer's feedback indicates satisfaction.
- $I_T(x) = 0.15$: 15% uncertainty leans towards satisfaction due to mixed comments.
- $I_F(x) = 0.1$: 10% uncertainty leans towards dissatisfaction due to delivery delays.
- $F_A(x) = 0.05$: 5% feedback explicitly indicates dissatisfaction.

This enables companies to better understand customer opinions by considering both the certainty and directionality of indeterminate feedback.

Proposition 2.36. *A Double-Valued Neutrosophic Logic (DVNL) can be transformed into a standard Neutrosophic Logic (NL) by redefining the indeterminacy membership values. Specifically, let A be a Double-Valued Neutrosophic Set (DVNS) characterized by:*

$$A = \{(x, T_A(x), I_T(x), I_F(x), F_A(x)) : x \in X\}.$$

The corresponding Neutrosophic Set A' is obtained as:

$$A' = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\},$$

where the total indeterminacy $I_A(x)$ is given by:

$$I_A(x) = I_T(x) + I_F(x).$$

Proof. Let A be a DVNS defined as:

$$A = \{(x, T_A(x), I_T(x), I_F(x), F_A(x)) : x \in X\}.$$

By definition, the total indeterminacy $I_A(x)$ in a Neutrosophic Logic framework combines the contributions from $I_T(x)$ and $I_F(x)$. Thus, we define:

$$I_A(x) = I_T(x) + I_F(x).$$

The transformed set A' becomes:

$$A' = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}.$$

The condition for a DVNS ensures:

$$0 \leq T_A(x) + I_T(x) + I_F(x) + F_A(x) \leq 4.$$

Since $I_A(x) = I_T(x) + I_F(x)$, the condition for A' simplifies to:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3,$$

which satisfies the standard Neutrosophic Logic framework. Therefore, A' is a valid Neutrosophic Set.

The transformation from DVNL to NL is achieved by aggregating $I_T(x)$ and $I_F(x)$ into a single indeterminacy membership value $I_A(x)$, preserving the consistency of the logical framework. □

Definition 2.37 (Upside-Down Transformation for DVNS)**.** For any $x \in X$, the Upside-Down Transformation U modifies the values as follows:

$$\begin{aligned} U(T_A(x)) &= F_A(x), \\ U(F_A(x)) &= T_A(x), \\ U(I_T(x)) &= I_F(x), \\ U(I_F(x)) &= I_T(x). \end{aligned}$$

Example 2.38 (Service Quality Evaluation)**.** Service Quality Evaluation is the process of assessing service performance based on customer expectations, perceptions, and satisfaction across various dimensions (cf. [86]).

Let $X = \{x_1, x_2, x_3\}$, where: x_1: Capability, x_2: Trustworthiness, and x_3: Price.

Define a DVNS A over X:

$$A = \{(x_1, 0.3, 0.4, 0.2, 0.1), (x_2, 0.5, 0.3, 0.1, 0.1), (x_3, 0.7, 0.2, 0.1, 0.0)\}.$$

Before UDL Transformation:

- For x_1: $T_A(x_1) = 0.3, I_T(x_1) = 0.4, I_F(x_1) = 0.2, F_A(x_1) = 0.1$.
- For x_2: $T_A(x_2) = 0.5, I_T(x_2) = 0.3, I_F(x_2) = 0.1, F_A(x_2) = 0.1$.
- For x_3: $T_A(x_3) = 0.7, I_T(x_3) = 0.2, I_F(x_3) = 0.1, F_A(x_3) = 0.0$.

After UDL Transformation:

- For x_1:
$$U(T_A(x_1)) = F_A(x_1) = 0.1, \quad U(F_A(x_1)) = T_A(x_1) = 0.3,$$
$$U(I_T(x_1)) = I_F(x_1) = 0.2, \quad U(I_F(x_1)) = I_T(x_1) = 0.4.$$
Result: $(x_1, 0.1, 0.2, 0.4, 0.3)$.
- For x_2:
$$U(T_A(x_2)) = F_A(x_2) = 0.1, \quad U(F_A(x_2)) = T_A(x_2) = 0.5,$$
$$U(I_T(x_2)) = I_F(x_2) = 0.1, \quad U(I_F(x_2)) = I_T(x_2) = 0.3.$$
Result: $(x_2, 0.1, 0.1, 0.3, 0.5)$.
- For x_3:
$$U(T_A(x_3)) = F_A(x_3) = 0.0, \quad U(F_A(x_3)) = T_A(x_3) = 0.7,$$
$$U(I_T(x_3)) = I_F(x_3) = 0.1, \quad U(I_F(x_3)) = I_T(x_3) = 0.2.$$
Result: $(x_3, 0.0, 0.1, 0.2, 0.7)$.

Observations:

- Truth (T_A) and falsity (F_A) are inverted, reflecting the essence of Upside-Down Logic.
- Indeterminacy leaning towards truth (I_T) and falsity (I_F) are swapped, altering the nuanced interpretation of uncertainty.
- This transformation highlights the flexibility and adaptability of DVN Logic under the influence of UDL, allowing for a richer representation of real-world ambiguity.

Theorem 2.39 (Stability of Membership Sum Constraint)**.** *The sum of the membership values remains within the allowable range* $[0, 4]$ *after the Upside-Down Transformation.*

Proof. Let $x \in X$. For the original Double-Valued Neutrosophic Set (DVNS) A, we have:

$$T_A(x) + I_T(x) + I_F(x) + F_A(x) \leq 4.$$

After applying the Upside-Down Transformation:

$$U(T_A(x)) = F_A(x), \quad U(F_A(x)) = T_A(x), \quad U(I_T(x)) = I_F(x), \quad U(I_F(x)) = I_T(x).$$

The transformed sum becomes:

$$U(T_A(x)) + U(I_T(x)) + U(I_F(x)) + U(F_A(x)) = F_A(x) + I_F(x) + I_T(x) + T_A(x).$$

Since the sum is a permutation of the original values:

$$F_A(x) + I_F(x) + I_T(x) + T_A(x) = T_A(x) + I_T(x) + I_F(x) + F_A(x).$$

Thus, the sum remains unchanged, and the constraint $0 \leq \text{Sum} \leq 4$ is preserved. □

Theorem 2.40 (Invertibility of the Upside-Down Transformation)**.** *The Upside-Down Transformation U is invertible.*

Proof. Define the transformation U as:

$$U(T_A(x)) = F_A(x), \quad U(F_A(x)) = T_A(x), \quad U(I_T(x)) = I_F(x), \quad U(I_F(x)) = I_T(x).$$

To invert U, apply the transformation again:

$$U(U(T_A(x))) = U(F_A(x)) = T_A(x),$$

$$U(U(F_A(x))) = U(T_A(x)) = F_A(x),$$
$$U(U(I_T(x))) = U(I_F(x)) = I_T(x),$$
$$U(U(I_F(x))) = U(I_T(x)) = I_F(x).$$

Thus, applying U twice returns the original values. Therefore, U is its own inverse, i.e., $U^2 = \text{Identity}$. □

Theorem 2.41 (Truth Preservation in Special Cases)**.** *If a DVNS has zero indeterminacy ($I_T(x) = I_F(x) = 0$), the transformation flips truth and falsity values but preserves their relationship.*

Proof. If $I_T(x) = I_F(x) = 0$, the DVNS simplifies to:

$$A = \{(x, T_A(x), 0, 0, F_A(x)) : x \in X\}.$$

Applying U:

$$U(A) = \{(x, F_A(x), 0, 0, T_A(x)) : x \in X\}.$$

Thus:

$$T_A(x) + F_A(x) \text{ (original sum)} = F_A(x) + T_A(x) \text{ (transformed sum)}.$$

Since the transformation merely swaps $T_A(x)$ and $F_A(x)$, the truth and falsity values are inverted while maintaining consistency. □

Upside-Down Logic is not limited to applications in Neutrosophic Logic or Double-Valued Neutrosophic Logic; it can be extended to various logical and set-based frameworks that handle uncertainty.

Although this paper does not delve into details, related concepts such as Neutrosophic OffLogic, Neutrosophic OverLogic, and Neutrosophic UnderLogic are also well-known. Neutrosophic OffLogic, for instance, operates not within the standard $[0, 1]$ interval but within an extended or alternative range of values, allowing greater flexibility in representing degrees of truth, indeterminacy, and falsity [18, 87, 91, 92, 101]. Similar to standard Neutrosophic Logic, these frameworks support the upside-down transformation between truth and falsity, enabling dynamic and adaptive reasoning under uncertain or contradictory conditions.

Question 2.42. Is it possible to mathematically define concepts of Upside-Down Logic in the context of set theory [50] or graph theory [28, 29]?

Question 2.43. Can Upside-Down Logic be applied beyond logic and set concepts? For example:

- In game theory [72], could it be used to model new equilibria or strategies by considering reversals of perspectives or situations?
- In belief revision [41], could it provide a framework for dynamically altering the truth or falsity of beliefs based on specific evidence or contexts?
- In quantum logic [31], could it be utilized to reverse states or transform the interpretation of measurement results?

2.7 Algorithm for Upside-Down Logic

An algorithm is a finite, step-by-step procedure designed to perform a specific task or solve a problem systematically and efficiently (cf. [22, 85]). Below, we present an algorithm for Upside-Down Logic.

Algorithm 1 Upside-Down Logic Algorithm

Require: A set of propositions $\mathcal{P}$, a context C, and a truth valuation function $T : \mathcal{P} \times C \to \{\text{True}, \text{False}, \text{Indeterminate}\}$

Ensure: Transformed propositions $U(A)$, transformed contexts $U(C)$, and updated truth values

Parse the input $\mathcal{P}$, C, and T. Define the transformation function U

for each $A \in \mathcal{P}$ **do**

  Compute $U(A)$ and $U(C)$

  Update truth values:

$$T(U(A), U(C)) = \begin{cases} \text{False}, & \text{if } T(A, C) = \text{True}, \\ \text{True}, & \text{if } T(A, C) = \text{False}, \\ \text{Indeterminate}, & \text{otherwise}. \end{cases}$$

end for

Apply U to logical connectives $(\wedge, \vee, \neg)$ and compute transformed truth tables

return $U(\mathcal{P})$, $U(C)$, and updated truth values

Theorem 2.44 (Correctness of the Upside-Down Logic Algorithm)**.** *The Upside-Down Logic algorithm transforms propositions and contexts according to the defined transformation function U, correctly updates truth values based on the rules of Upside-Down Logic, and preserves logical consistency.*

Proof. The algorithm processes the input as follows:

1. Initialization: The input sets $\mathcal{P}$, C, and the truth valuation function T are parsed correctly. The transformation U is well-defined and invertible, ensuring valid inputs and outputs.

2. Transformation of Propositions and Contexts: For each $A \in \mathcal{P}$, the algorithm computes $U(A)$ and $U(C)$. These transformations are consistent with the Upside-Down Logic framework, ensuring valid outputs $U(\mathcal{P})$ and $U(C)$.

3. Truth Value Update: The algorithm updates truth values as:

$$T(U(A), U(C)) = \begin{cases} \text{False}, & \text{if } T(A, C) = \text{True}, \\ \text{True}, & \text{if } T(A, C) = \text{False}, \\ \text{Indeterminate}, & \text{otherwise.} \end{cases}$$

This ensures that True becomes False, False becomes True, and Indeterminate remains unchanged, respecting the rules of Upside-Down Logic.

4. Adjustment of Logical Connectives: Logical connectives $(\wedge, \vee, \neg)$ are transformed consistently under U, maintaining logical relationships within $U(C)$.

5. Logical Consistency: Since U is invertible and operates independently on propositions and contexts, logical consistency is preserved throughout the transformation process.

The algorithm correctly implements the rules of Upside-Down Logic, updates all truth values systematically, and ensures logical consistency in the transformed system. Therefore, the algorithm is correct. □

Definition 2.45. (cf. [75, 85]) The *Total Time Complexity* of an algorithm is defined as the sum of the time required to execute each step of the algorithm, expressed as a function of the input size. If an algorithm involves multiple steps or operations, the total time complexity is determined by the maximum time required for the most time-consuming operation.

Formally, let $T(n, m)$ be the time complexity as a function of input sizes n and m. The total time complexity is:

$$T(n, m) = \max(T_{\text{operation1}}(n, m), T_{\text{operation2}}(n, m), \dots, T_{\text{operationk}}(n, m)),$$

where n is the size of the set of propositions and m is the size or complexity of the context.

Definition 2.46. (cf. [75, 85]) The *Space Complexity* of an algorithm is the total amount of memory required to execute the algorithm, expressed as a function of the input size. This includes:

- The *input space*, which depends on the size of the input n, m,
- The *auxiliary space*, which includes temporary variables, data structures, or storage used during computation.

Formally, let $S(n, m)$ be the space complexity as a function of input sizes n and m. The total space complexity is:

$$S(n, m) = S_{\text{input}}(n, m) + S_{\text{auxiliary}}(n, m).$$

Definition 2.47. (cf. [75, 85]) *Big-O notation* is a mathematical concept used to describe the upper bound of the time or space complexity of an algorithm. Let $f(n)$ and $g(n)$ be functions that map non-negative integers to non-negative real numbers. We say:

$$f(n) \in O(g(n))$$

if there exist constants $c > 0$ and $n_0 \geq 0$ such that:

$$f(n) \leq c \cdot g(n) \quad \text{for all } n \geq n_0.$$

Theorem 2.48 (Time and Space Complexity of Upside-Down Logic Algorithm)**.** *In the Upside-Down Logic algorithm, the following complexities hold: Total Time Complexity: $O(n + m)$.*
Space Complexity: $O(n + m)$.

Proof. Step 1: Transformation of Propositions
For a set of n propositions $\mathcal{P}$, each proposition $A \in \mathcal{P}$ is transformed using the Upside-Down transformation U. Since the transformation involves a single operation per proposition, the complexity is $O(n)$.

Step 2: Transformation of Context
The transformation of the context C involves modifying m parameters (e.g., spatial, temporal, semantic attributes). Each parameter requires a constant amount of work, resulting in a complexity of $O(m)$.

Step 3: Truth Value Updates
Each proposition $A \in \mathcal{P}$ is evaluated within the transformed context $U(C)$. The evaluation of truth values requires m operations per context, resulting in a complexity of $O(m)$ for each proposition. Since there are n propositions, the total complexity for truth value updates is $O(n \cdot m)$.

Step 4: Logical Connective Adjustment
Logical connectives ($\wedge, \vee, \neg$) associated with each proposition are adjusted once per proposition. This results in a complexity of $O(n)$.

Total Time Complexity:
Summing the complexities of all steps:

$$\text{Total Time Complexity} = O(n) + O(m) + O(n \cdot m) + O(n).$$

The term $O(n \cdot m)$ dominates for large n and m. However, if m is constant or small relative to n, the total complexity simplifies to $O(n + m)$.

Space Complexity:
The algorithm requires memory to store:

- The transformed propositions $U(\mathcal{P})$ ($O(n)$),
- The transformed context $U(C)$ ($O(m)$),
- The updated truth values ($O(n \cdot m)$).

Combining these, the total space complexity is $O(n + m)$.

Thus, the stated complexities are proven. □

3 Contextual Upside-Down Logic

In this section, We introduce a derivative logic called *Contextual Upside-Down Logic*, which extends Upside-Down Logic by incorporating a contextual transformation function that not only flips truth values but also adjusts the logical connectives based on context.

3.1 Formal Definition of Contextual Upside-Down Logic

The definition of Contextual Upside-Down Logic is provided below. In a highly intuitive sense, it is about defining a system that controls "differences in context" using different logical operations. To put it boldly, it can be seen as a definition aimed at reducing the probability of Upside-Down occurrences caused by various factors. There is no significant operational difference from Upside-Down Logic; the only distinction is that the contextual transformation function is explicitly defined.

Definition 3.1 (Contextual Upside-Down Logic)**.** A *Contextual Upside-Down Logic* is a logical system $\mathcal{M}''$ derived from a base logical system $\mathcal{M}$ by introducing a context-dependent transformation U_C such that:

1. For any proposition $A \in \mathcal{P}$ with truth value $v(A, C)$ in context C, there exists a transformed proposition $U_C(A)$ and transformed context $U_C(C)$ where:
   - *Contextual Falsification*: If $v(A, C) = \text{True}$, then $v(U_C(A), U_C(C)) = \text{False}$.
   - *Contextual Truthification*: If $v(A, C) = \text{False}$, then $v(U_C(A), U_C(C)) = \text{True}$.

2. The logical connectives are adjusted according to the context transformation, allowing for different logical operations in $U_C(C)$.

3. The transformation U_C is well-defined, invertible, and consistent within the logical system $\mathcal{M}''$.

Remark 3.2. The comparison with General Upside-Down Logic and notable points are described below.

- *Context Dependence*: General Upside-Down Logic applies transformations U globally, altering truth values uniformly across all contexts. In contrast, Contextual Upside-Down Logic introduces a transformation U_C that is explicitly dependent on the context C, allowing for adjustments that are sensitive to specific cultural, semantic, or situational factors.
- *Adjustment of Logical Connectives*: In Contextual Upside-Down Logic, not only are truth values flipped based on context, but logical connectives are also adjusted. This means that operations like "and," "or," "implies," etc., may have different interpretations in different contexts, affecting the outcome of logical evaluations.
- *Transformation Scope*: While Upside-Down Logic applies transformations uniformly, Contextual Upside-Down Logic allows transformations to vary between contexts, leading to a more nuanced and accurate modeling of propositions in context-dependent scenarios.

3.2 Some Example of Contextual Upside-Down Logic

In this subsection, we explore some examples of Contextual Upside-Down Logic.

Example 3.3 (Cultural Interpretation of Silence)**.** Let A: "Silence implies agreement."

In context C_{Western}:

In some Western contexts, silence during a discussion can be interpreted as agreement or consent:

$$v(A, C_{\text{Western}}) = \text{True}.$$

Contextual Transformation:

In Japanese cultural context $U_C(C_{\text{Japanese}})$, silence often carries different meanings, such as disagreement, contemplation, or polite refusal.

Applying the context-dependent transformation U_C, we adjust both the proposition and the logical connective "implies":

$$U_C(A) : \text{"Silence does not imply agreement."}$$

The logical connective "implies" is reinterpreted based on the cultural context.

Updated Truth Value:

In the transformed context:

$$v(U_C(A), U_C(C_{\text{Japanese}})) = \text{True}.$$

This example demonstrates how Contextual Upside-Down Logic adjusts both the proposition and the logical connective based on cultural context, differing from general Upside-Down Logic by its context-sensitive approach.

Example 3.4 (Double Negatives in Language)**.** Double Negative refers to the use of two negative terms in a sentence, which can create emphasis or reverse the intended meaning(cf. [9]).

Let A: ”He didn’t do nothing.”

In context $C_{\text{Standard English}}$:

In Standard English, double negatives are typically interpreted as a positive statement due to the logical rule that two negatives make a positive:

$$v(A, C_{\text{Standard English}}) = \text{True}.$$

Contextual Transformation:

In certain dialects or colloquial contexts $U_C(C_{\text{Dialectal English}})$, double negatives are used for emphasis and still convey a negative meaning.

Applying the context-dependent transformation U_C, the logical connective (negation) is adjusted:

$$U_C(A) : \text{”He did not do anything.”}$$

Updated Truth Value:

In the transformed context:

$$v(U_C(A), U_C(C_{\text{Dialectal English}})) = \text{False}.$$

This example shows how Contextual Upside-Down Logic adjusts the interpretation of logical connectives based on linguistic context, leading to different truth values.

Example 3.5 (Logical Connectives in Cultural Context)**.** Let A: ”He is smart and hardworking.”

In context $C_{\text{Culture A}}$:

In Culture A, the conjunction ”and” is used to combine two positive attributes straightforwardly:

$$v(A, C_{\text{Culture A}}) = \text{True}.$$

Contextual Transformation:

In Culture B, where modesty is highly valued, praising someone with multiple positive attributes may be seen as excessive or may imply that one attribute detracts from the other.

Applying the context-dependent transformation U_C, the logical connective ”and” is adjusted to reflect this cultural nuance:

$$U_C(A) : \text{”He is smart but not necessarily hardworking.”}$$

Or the ”and” might be interpreted as an exclusive ”or”:

$$U_C(A) : \text{”He is smart or hardworking (but not both).”}$$

Updated Truth Value:

In the transformed context:

$$v(U_C(A), U_C(C_{\text{Culture B}})) = \text{Uncertain or False}.$$

This example illustrates how the interpretation of logical connectives like "and" can change based on cultural context, a feature captured by Contextual Upside-Down Logic.

Example 3.6 (Indirect Communication in Japanese Culture)**.** Indirect Communication in Japanese Culture emphasizes subtlety, non-verbal cues, and ambiguity, often prioritizing social harmony and avoiding direct confrontation or disagreement (cf. [30, 68]).

Let A: "He is refusing the request."

In context $C_{\textbf{Direct Communication}}$:

In a culture that values direct communication, saying "I will think about it" suggests consideration, implying that the person may accept the request:

$$v(A, C_{\text{Direct Communication}}) = \text{False}.$$

Contextual Transformation:

In Japanese culture $U_C(C_{\text{Japanese}})$, the phrase "*Kangaete okimasu*" ("I will think about it") is often a polite way to decline a request.

Applying the context-dependent transformation U_C, we adjust the proposition and interpret the logical implications differently:

$$U_C(A) : \text{"He is politely declining the request."}$$

Updated Truth Value:

In the transformed context:

$$v(U_C(A), U_C(C_{\text{Japanese}})) = \text{True}.$$

The transformation demonstrates how Contextual Upside-Down Logic captures the nuances of indirect communication by adjusting the proposition and its interpretation based on cultural context.

Example 3.7 (Interpretation of "Yes" in Different Contexts)**.** Let A: "She agrees with the proposal."

In context $C_{\textbf{General}}$:

In many cultures, saying "yes" directly signifies agreement:

$$v(A, C_{\text{General}}) = \text{True}.$$

Contextual Transformation:

In Japanese culture $U_C(C_{\text{Japanese}})$, saying "*Hai*" can sometimes mean "I hear you" or "I acknowledge what you are saying," without implying agreement.

Applying the context-dependent transformation U_C, we adjust the proposition:

$$U_C(A) : \text{"She acknowledges the proposal but may not agree with it."}$$

Updated Truth Value:

In the transformed context:

$$v(U_C(A), U_C(C_{\text{Japanese}})) = \text{Uncertain or False.}$$

This example highlights how Contextual Upside-Down Logic accounts for cultural differences in interpreting affirmations, adjusting both the proposition and its truth value accordingly.

Example 3.8 (Concept of "Honne" and "Tatemae" in Japanese Culture)**.** Let A: "He is expressing his true opinions."

In context $C_{\text{Individualistic}}$:

In an individualistic culture where personal expression is encouraged, the proposition is likely considered true:

$$v(A, C_{\text{Individualistic}}) = \text{True.}$$

Contextual Transformation:

In Japanese culture $U_C(C_{\text{Japanese}})$, the concepts of *Honne* (true feelings) and *Tatemae* (public facade) play a significant role. Individuals often express *Tatemae* to maintain social harmony(cf. [17, 60]).

Applying the context-dependent transformation U_C:

$$U_C(A) : \text{"He is expressing what is expected socially, not his true opinions."}$$

Updated Truth Value:

In the transformed context:

$$v(U_C(A), U_C(C_{\text{Japanese}})) = \text{False.}$$

This demonstrates how Contextual Upside-Down Logic captures the impact of cultural concepts on truth values and the interpretation of propositions.

Example 3.9 (Adjusting Logical Connectives Based on Context)**.** Let A: "Taking risks leads to success."

In context $C_{\text{Entrepreneurial}}$:

In an entrepreneurial context where risk-taking is valued, the proposition is considered true:

$$v(A, C_{\text{Entrepreneurial}}) = \text{True.}$$

Contextual Transformation:

In a context where stability is prioritized, such as in certain traditional cultures $U_C(C_{\text{Traditional}})$, the logical connective "leads to" may be interpreted differently, perhaps even reversed.

Applying the context-dependent transformation U_C:

$$U_C(A) : \text{"Taking risks leads to failure."}$$

Updated Truth Value:

In the transformed context:

$$v(U_C(A), U_C(C_{\text{Traditional}})) = \text{True}.$$

This example shows how Contextual Upside-Down Logic adjusts both the proposition and the logical connective based on context, altering the truth value accordingly.

Remark 3.10. In these examples, Contextual Upside-Down Logic differs from general Upside-Down Logic by:

- **Context-Specific Transformations**: Adjustments are made based on specific contexts rather than applying a uniform transformation across all contexts.
- **Adjustment of Logical Connectives**: Logical connectives are reinterpreted according to the context, affecting how propositions are connected and evaluated.
- **Nuanced Truth Values**: Truth values may become uncertain or require more nuanced interpretation due to context-dependent meanings.

3.3 Some Basic Theorem of Contextual Upside-Down Logic

In this subsection, we consider some basic theorems of Contextual Upside-Down Logic. The following theorems hold.

Definition 3.11 (Bijective Function)**.** A function $f : X \to Y$ is called *bijective* if it is both injective (one-to-one) and surjective (onto). This means:

- f is **injective**: For all $x_1, x_2 \in X$, if $f(x_1) = f(x_2)$, then $x_1 = x_2$.
- f is **surjective**: For every $y \in Y$, there exists at least one $x \in X$ such that $f(x) = y$.

Thus, every element in Y is mapped to by exactly one element in X.

Definition 3.12 (Invertible Function)**.** A function $f : X \to Y$ is called *invertible* if there exists a function $g : Y \to X$ such that:

$$g(f(x)) = x \quad \text{for all } x \in X, \quad \text{and} \quad f(g(y)) = y \quad \text{for all } y \in Y.$$

The function g is called the *inverse* of f, and is denoted by f^{-1}.

Theorem 3.13 (Contextual Invertibility)**.** *In Contextual Upside-Down Logic, the transformation U_C is invertible if and only if the context mapping $U_C : C \to U_C(C)$ is bijective.*

Proof. If U_C is invertible, there exists a U_C^{-1} such that:

$$U_C^{-1}(U_C(A)) = A, \quad U_C^{-1}(U_C(C)) = C.$$

This requires that U_C is a bijection on both propositions and contexts.

Conversely, if U_C is bijective, then it has an inverse function, and the transformation is invertible. □

Theorem 3.14 (Consistency Preservation)**.** *If the base logic $\mathcal{M}$ is consistent, and the transformation U_C is consistent, then the derived Contextual Upside-Down Logic $\mathcal{M}''$ is consistent.*

Proof. Since U_C transforms propositions and contexts without introducing contradictions, and $\mathcal{M}$ is consistent, any deductions made in $\mathcal{M}''$ will also be consistent. □

3.4 Algorithm for Contextual Upside-Down Transformation

We present an algorithm that, given a proposition A and a context C, computes the transformed proposition $U_C(A)$ and the transformed context $U_C(C)$.

Algorithm 2 Contextual Upside-Down Transformation Algorithm

Require: Proposition A, Context C, Valuation Function v
Ensure: Transformed Proposition $U_C(A)$, Transformed Context $U_C(C)$, Updated Truth Value $v(U_C(A), U_C(C))$

Compute the truth value $v(A, C)$
if $v(A, C) = \text{True}$ **then**
  Apply a context transformation function $T_C : C \to U_C(C)$
  Define $U_C(C) = T_C(C)$ such that $v(A, U_C(C)) = \text{False}$
else if $v(A, C) = \text{False}$ **then**
  Apply a context transformation function $T_C : C \to U_C(C)$
  Define $U_C(C) = T_C(C)$ such that $v(A, U_C(C)) = \text{True}$
end if
Define $U_C(A) = A$
Adjust the logical connectives within A to reflect changes in $U_C(C)$, if necessary
return $U_C(A), U_C(C), v(U_C(A), U_C(C))$

Theorem 3.15 (Correctness of the Algorithm)**.** *The Contextual Upside-Down Transformation Algorithm correctly computes the transformed proposition and context such that the truth value of A is inverted in the transformed context, satisfying the definitions of Contextual Upside-Down Logic.*

Proof. Consider the two cases:

1. If $v(A, C) = \text{True}$, the algorithm applies a context transformation T_C such that $U_C(C)$ makes A false. By definition of Contextual Falsification, $v(U_C(A), U_C(C)) = \text{False}$.
2. If $v(A, C) = \text{False}$, the algorithm applies a context transformation T_C such that $U_C(C)$ makes A true. By definition of Contextual Truthification, $v(U_C(A), U_C(C)) = \text{True}$.

Thus, the algorithm adheres to the principles of Contextual Upside-Down Logic. □

Theorem 3.16 (Time and Space Complexity of the Contextual Upside-Down Transformation Algorithm)**.** *Let n represent the size of the proposition A (e.g., the number of symbols or logical operators), and m represent the complexity of the context C. The following hold for the Contextual Upside-Down Transformation Algorithm:*

1. *The total time complexity is $O(n + m)$.*

2. *The total space complexity is* $O(n+m)$.

Proof. To prove the claims, we analyze each step of the algorithm.

1. *Truth Value Computation*: This step evaluates $v(A, C)$, which involves traversing A and assessing its logical structure within the context C.
   - Traversing A contributes $O(n)$.
   - Accessing contextual information for C contributes $O(m)$.

   Therefore, this step has a complexity of $O(n+m)$.
2. *Context Transformation*: The context C is transformed into $U_C(C)$ using a predefined transformation function T_C.
   - Since this operation is independent of A, the complexity depends solely on C and is $O(m)$.
3. *Logical Connective Adjustment*: Adjusting logical connectives within A requires traversing the structure of A.
   - This contributes $O(n)$.
4. *Total Time Complexity*: Summing up the complexities of the above steps, the total time complexity is:

$$O(n+m) + O(m) + O(n) = O(n+m).$$

5. *Space Complexity*: The algorithm requires space to store the transformed proposition $U_C(A)$ and the transformed context $U_C(C)$.
   - The storage for A contributes $O(n)$.
   - The storage for C contributes $O(m)$.

   Thus, the total space complexity is:

$$O(n+m).$$

This completes the proof of the theorem. □

Example 3.17 (Ambiguity in Language Translation using the Algorithm)**.** We consider following example.

- *Proposition*: A: ”He is cool.”
- *Context*: C: American English, where ”cool” means ”good” or ”impressive.”
- *Valuation*: $v(A, C) = \text{True}$.

Transformation: Transform the context to $U_C(C)$: British English, where ”cool” might mean ”cold” or emotionally distant. $v(U_C(A), U_C(C)) = \text{False}$.

Thus, the algorithm successfully inverts the truth value via context transformation.

4 Framework of Indeterm-Upside-Down Logic

We propose a new framework called Indeterm-Upside-Down Logic. This logic is designed to represent real-world phenomena where Indeterminate either decreases or increases.

4.1 Definition and Mathematical Formulation

We examine the definition and mathematical formulation of Indeterm-Upside-Down Logic.

Notation 4.1. *Let $\mathcal{P}$ be a set of propositions, and let C be a set of contexts. Let $T : \mathcal{P}\times C \to \{True, False, Indeterminate\}$ be a truth valuation function that assigns a truth value to each proposition-context pair.*

The following four transformations are defined within the framework of Indeterm-Upside-Down Logic:

- Indeterminate-to-True Transformation: Transforms indeterminate propositions into true ones when evidence strongly supports their truthfulness.
- Indeterminate-to-False Transformation: Converts indeterminate propositions to false when new evidence contradicts their truthfulness.
- True-to-Indeterminate Transformation: Shifts propositions from true to indeterminate when supporting evidence becomes unreliable or insufficient.
- False-to-Indeterminate Transformation: Transforms propositions from false to indeterminate when evidence previously supporting falsity becomes inconclusive.

The framework of Indeterm-Upside-Down Logic, including its definitions and brief examples, is presented below.

Definition 4.2 (Indeterminate-to-True Transformation)**.** Let $A \in \mathcal{P}$, and suppose $T(A, C) = \text{Indeterminate}$. The transformation U_{IT} (Indeterminate-to-True) changes the truth value as follows:

$$T(U_{\text{IT}}(A), U_{\text{IT}}(C)) = \text{True}.$$

This transformation is applied if additional evidence or contextual information **supports the truthfulness** of the proposition.

Example 4.3 (Diagnosing a Medical Condition)**.** Let A: "The patient has a specific medical condition."

Initial Context C: The initial test results are inconclusive, making the truth value of A indeterminate:

$$T(A, C) = \text{Indeterminate}.$$

Context Transformation $U_{\text{IT}}(C)$: New diagnostic evidence, such as advanced imaging or biomarkers, strongly supports the presence of the condition:

$$T(U_{\text{IT}}(A), U_{\text{IT}}(C)) = \text{True}.$$

Interpretation: As new evidence emerges, the truth value of the proposition A shifts from indeterminate to true, allowing for a definitive diagnosis.

Example 4.4 (Japanese Linguistic Ambiguity)**.** Let E: "*Sore wa chotto...*" means "The speaker is refusing the request."

Initial Context C: In a literal interpretation, the phrase appears hesitant and non-committal:

$$T(E, C) = \text{Indeterminate}.$$

Context Transformation $U_{\text{IT}}(C)$: Understanding cultural subtleties in context $U_{\text{IT}}(C)$, the phrase is recognized as a polite refusal:

$$T(U_{\text{IT}}(E), U_{\text{IT}}(C)) = \text{True}.$$

Interpretation: Cultural context shifts the interpretation of the ambiguous phrase to convey a clear refusal.

Definition 4.5 (Indeterminate-to-False Transformation)**.** Let $A \in \mathcal{P}$, and suppose $T(A, C) =$ Indeterminate. The transformation U_{IF} (Indeterminate-to-False) changes the truth value as follows:

$$T(U_{\text{IF}}(A), U_{\text{IF}}(C)) = \text{False}.$$

This transformation is applied if additional evidence or contextual information **contradicts the truthfulness** of the proposition.

Example 4.6 (Weather Prediction)**.** Let B: "It will rain tomorrow."

Initial Context C: Early weather models are ambiguous, leading to an indeterminate truth value for B:

$$T(B, C) = \text{Indeterminate}.$$

Context Transformation $U_{\text{IF}}(C)$: Updated weather data shows clear skies, strongly contradicting the likelihood of rain:

$$T(U_{\text{IF}}(B), U_{\text{IF}}(C)) = \text{False}.$$

Interpretation: As more reliable weather information becomes available, the proposition B transitions from indeterminate to false.

Definition 4.7 (True-to-Indeterminate Transformation)**.** Let $B \in \mathcal{P}$, and suppose $T(B, C) =$ True. The transformation U_{TI} (True-to-Indeterminate) changes the truth value as follows:

$$T(U_{\text{TI}}(B), U_{\text{TI}}(C)) = \text{Indeterminate}.$$

This transformation is applied if evidence or contextual information that previously supported the truth of B becomes unreliable or insufficient.

Example 4.8 (Legal Testimony)**.** Let C: "The witness's statement is truthful."

Initial Context C: Based on initial corroboration, the statement is considered true:

$$T(C, C) = \text{True}.$$

Context Transformation $U_{\text{TI}}(C)$: New contradictory evidence or inconsistencies emerge, casting doubt on the testimony:

$$T(U_{\text{TI}}(C), U_{\text{TI}}(C)) = \text{Indeterminate}.$$

Interpretation: The emergence of conflicting information shifts the truth value of C from true to indeterminate, reflecting the uncertainty surrounding the testimony.

Example 4.9 (Scientific Consensus)**.** Let G: "A particular medicine is effective for treating a disease."

Initial Context C: Clinical trials initially support the efficacy of the medicine:

$$T(G, C) = \text{True}.$$

Context Transformation $U_{\text{TI}}(C)$: New studies produce mixed results, creating uncertainty:

$$T(U_{\text{TI}}(G), U_{\text{TI}}(C)) = \text{Indeterminate}.$$

Interpretation: Shifting evidence impacts the certainty of scientific claims, transitioning from true to indeterminate.

Definition 4.10 (False-to-Indeterminate Transformation)**.** Let $C \in \mathcal{P}$, and suppose $T(C, C) = \text{False}$. The transformation U_{FI} (False-to-Indeterminate) changes the truth value as follows:

$$T(U_{\text{FI}}(C), U_{\text{FI}}(C)) = \text{Indeterminate}.$$

This transformation is applied if evidence or contextual information that previously supported the falsity of C becomes unreliable or insufficient.

Example 4.11 (Product Safety Assessment)**.** Let D: "The product is unsafe for use."

Initial Context C: Initial tests indicate that the product is unsafe:

$$T(D, C) = \text{False}.$$

Context Transformation $U_{\text{FI}}(C)$: Additional testing and revised safety guidelines introduce ambiguity regarding its safety:

$$T(U_{\text{FI}}(D), U_{\text{FI}}(C)) = \text{Indeterminate}.$$

Interpretation: The shift from false to indeterminate reflects new uncertainty about the safety of the product under revised conditions.

4.2 Neutrosophic Logic Representation

In Neutrosophic Logic, each proposition $A \in \mathcal{P}$ is assigned a truth value $v(A) = (T, I, F)$, where $T, I, F \in [0, 1]$ represent the degrees of truth, indeterminacy, and falsity, respectively. The transformations can be described mathematically as follows:

Definition 4.12. Given $v(A) = (T, I, F)$ with $I > 0$ and evidence supporting truth:

$$v(U_{\text{IT}}(A)) = (T + I, 0, F).$$

Definition 4.13. Given $v(A) = (T, I, F)$ with $I > 0$ and evidence contradicting truth:

$$v(U_{\text{IF}}(A)) = (T, 0, F + I).$$

Definition 4.14. Given $v(B) = (T, I, F)$ with $T > 0$ and emerging ambiguity:

$$v(U_{\text{TI}}(B)) = (0, T + I, F).$$

Definition 4.15. Given $v(C) = (T, I, F)$ with $F > 0$ and emerging ambiguity:

$$v(U_{\text{FI}}(C)) = (T, I + F, 0).$$

Remark 4.16.

1. **Preservation of Total Degree:** All transformations preserve the sum of the components:

$$T + I + F = T' + I' + F'.$$

2. **Consistency:** The transformations satisfy the constraints of Neutrosophic Logic, where $T, I, F \in [0, 1]$ and $T + I + F \leq 1$.

4.3 Mathematical Basic Theorems in Indeterm-Upside-Down Logic

This subsection presents formal theorems within the framework of Indeterm-Upside-Down Logic.

Theorem 4.17 (Preservation of Total Truth Value)**.** *For any proposition $A \in \mathcal{P}$ with a truth value $T(A, C) \in \{True, False, Indeterminate\}$, every transformation $U_{IT}, U_{IF}, U_{TI}, U_{FI}$ preserves the total degree of truth, falsity, and indeterminacy. That is:*

$$T + I + F = T' + I' + F',$$

where (T, I, F) and (T', I', F') represent the truth value components before and after the transformation.

Proof. Consider the four transformations:

1. **Indeterminate-to-True ($U_{\mathbf{IT}}$):**

$$v(A) = (T, I, F) \quad \rightarrow \quad v(U_{\mathrm{IT}}(A)) = (T + I, 0, F).$$

   Clearly, $T + I + F = (T + I) + 0 + F$.

2. **Indeterminate-to-False ($U_{\mathbf{IF}}$):**

$$v(A) = (T, I, F) \quad \rightarrow \quad v(U_{\mathrm{IF}}(A)) = (T, 0, F + I).$$

   Similarly, $T + I + F = T + 0 + (F + I)$.

3. **True-to-Indeterminate ($U_{\mathbf{TI}}$):**

$$v(A) = (T, I, F) \quad \rightarrow \quad v(U_{\mathrm{TI}}(A)) = (0, T + I, F).$$

   Here, $T + I + F = 0 + (T + I) + F$.

4. **False-to-Indeterminate ($U_{\mathbf{FI}}$):**

$$v(A) = (T, I, F) \quad \rightarrow \quad v(U_{\mathrm{FI}}(A)) = (T, I + F, 0).$$

   Finally, $T + I + F = T + (I + F) + 0$.

In each case, the total sum $T + I + F$ remains invariant. Thus, the theorem holds. □

Theorem 4.18 (Involutivity of U_{TI} and U_{FI})**.** *The transformations U_{TI} (True-to-Indeterminate) and U_{FI} (False-to-Indeterminate) are involutive. That is:*

$$U_{TI}(U_{TI}(A)) = A, \quad U_{FI}(U_{FI}(A)) = A.$$

Proof. Let $v(A) = (T, I, F)$.

1. **True-to-Indeterminate ($U_{\mathbf{TI}}$):**

$$v(A) = (T, I, F) \quad \rightarrow \quad v(U_{\mathrm{TI}}(A)) = (0, T + I, F).$$

   Applying U_{TI} again:

$$v(U_{\mathrm{TI}}(U_{\mathrm{TI}}(A))) = (T, I, F),$$

   which is identical to the original truth value.

2. **False-to-Indeterminate ($U_{\mathbf{FI}}$):**

$$v(A) = (T, I, F) \quad \rightarrow \quad v(U_{\mathrm{FI}}(A)) = (T, I + F, 0).$$

   Applying U_{FI} again:

$$v(U_{\mathrm{FI}}(U_{\mathrm{FI}}(A))) = (T, I, F),$$

   again identical to the original truth value.

Thus, U_{TI} and U_{FI} are involutive. □

Theorem 4.19 (Correctness of U_{IT} and U_{IF})**.** *For any $A \in \mathcal{P}$ with $T(A, C)$ = Indeterminate, the transformations U_{IT} (Indeterminate-to-True) and U_{IF} (Indeterminate-to-False) are consistent with the supporting or contradicting evidence E. That is:*

$$U_{IT} \textit{ is applied if and only if } E \textit{ supports } A,$$

$$U_{IF} \textit{ is applied if and only if } E \textit{ contradicts } A.$$

Proof. By definition:

1. **Indeterminate-to-True ($U_{\mathbf{IT}}$):** This transformation is applied only if evidence E supports A. Let E be a set of observations or data such that:
$$P(A \mid E) > 0.5,$$
where $P(A \mid E)$ is the probability of A being true given E. In this case:
$$T(U_{\mathrm{IT}}(A), U_{\mathrm{IT}}(C)) = \text{True}.$$

2. **Indeterminate-to-False ($U_{\mathbf{IF}}$):** This transformation is applied only if evidence E contradicts A. Let $P(A \mid E) < 0.5$. In this case:
$$T(U_{\mathrm{IF}}(A), U_{\mathrm{IF}}(C)) = \text{False}.$$

The correctness of the transformations follows from their alignment with the evidence E. □

4.4 Real-Life Examples: Combining Neutrosophic Logic with Indeterm-Upside-Down Logic

When Neutrosophic Logic is combined with Indeterm-Upside-Down Logic, we can model scenarios where ambiguity dynamically shifts, either increasing or decreasing, based on contextual evidence or interpretation. Below are some concrete real-life examples that illustrate these transformations.

Example 4.20 (Medical Diagnosis)**.** Consider a proposition A: ”The patient has Disease X.”

Initial Neutrosophic Value: $v(A) = (0.4, 0.5, 0.1)$, where:

- $T = 0.4$: Moderate evidence supports the diagnosis.
- $I = 0.5$: There is significant ambiguity due to inconclusive test results.
- $F = 0.1$: Minimal evidence contradicts the diagnosis.

Transformation $U_{\mathbf{IT}}$ (Indeterminate to Truth): New test results strongly support the diagnosis, reducing ambiguity. The updated value is:

$$v(U_{\mathrm{IT}}(A)) = (0.4 + 0.5, 0, 0.1) = (0.9, 0, 0.1).$$

Interpretation: The ambiguity is resolved in favor of the diagnosis, leaving a highly probable truth.

Transformation $U_{\mathbf{IF}}$ (Indeterminate to Falsity): Alternatively, if the new evidence contradicts the diagnosis:

$$v(U_{\mathrm{IF}}(A)) = (0.4, 0, 0.1 + 0.5) = (0.4, 0, 0.6).$$

Interpretation: Ambiguity is resolved against the diagnosis, indicating a stronger falsity.

Example 4.21 (Weather Prediction)**.** Consider a proposition B: ”It will rain tomorrow.”

Initial Neutrosophic Value: $v(B) = (0.6, 0.3, 0.1)$, where:

- $T = 0.6$: Moderate evidence (e.g., weather models) suggests rain.
- $I = 0.3$: Uncertainty due to fluctuating weather conditions.
- $F = 0.1$: Weak evidence opposes rain.

Transformation $U_{\mathbf{TI}}$ (Truth to Indeterminate): Unforeseen atmospheric changes increase uncertainty, reducing confidence in the prediction:

$$v(U_{\mathrm{TI}}(B)) = (0, 0.6 + 0.3, 0.1) = (0, 0.9, 0.1).$$

Interpretation: The truth value diminishes, and ambiguity dominates.

Transformation $U_{\mathbf{FI}}$ (Falsity to Indeterminate): If evidence opposing rain becomes unclear, ambiguity increases:

$$v(U_{\mathrm{FI}}(B)) = (0.6, 0.3 + 0.1, 0) = (0.6, 0.4, 0).$$

Interpretation: Falsity diminishes, and ambiguity increases while truth remains steady.

Example 4.22 (Consumer Product Reviews)**.** Consider a proposition C: "The product is high-quality."

Initial Neutrosophic Value: $v(C) = (0.7, 0.2, 0.1)$, where:

- $T = 0.7$: Most reviews are positive.
- $I = 0.2$: Some reviews lack clarity.
- $F = 0.1$: A minority of reviews are negative.

Transformation $U_{\mathbf{IT}}$ (Indeterminate to Truth): Additional positive reviews clarify doubts, increasing the truth value:

$$v(U_{\mathrm{IT}}(C)) = (0.7 + 0.2, 0, 0.1) = (0.9, 0, 0.1).$$

Transformation $U_{\mathbf{TI}}$ (Truth to Indeterminate): If conflicting reviews emerge, ambiguity grows:

$$v(U_{\mathrm{TI}}(C)) = (0, 0.7 + 0.2, 0.1) = (0, 0.9, 0.1).$$

Remark 4.23. These examples demonstrate how combining Neutrosophic Logic with Indeterm-Upside-Down Logic provides a nuanced framework for modeling dynamic changes in truth, indeterminacy, and falsity. This approach is particularly suited for real-world scenarios where evidence evolves over time.

4.5 Algorithm for Indeterm-Upside-Down Logic

We propose an algorithmic framework for Indeterm-Upside-Down Logic, incorporating the four transformations: Indeterminate-to-True, Indeterminate-to-False, True-to-Indeterminate, and False-to-Indeterminate. The algorithm determines the transformation to apply based on the current truth value, context, and available evidence.

Theorem 4.24 (Correctness of Algorithm)**.** *Algorithm 3 correctly applies the appropriate transformation based on the given inputs.*

Proof. The algorithm handles all cases as defined in the framework of Indeterm-Upside-Down Logic:

- If $T(A, C) =$ Indeterminate, it applies U_{IT} if evidence supports A, or U_{IF} if evidence contradicts A.
- If $T(A, C) =$ True, it applies U_{TI} if evidence becomes unreliable, transitioning the truth value to Indeterminate.
- If $T(A, C) =$ False, it applies U_{FI} if evidence becomes unreliable, transitioning the truth value to Indeterminate.

These conditions align with the definitions of Indeterm-Upside-Down Logic transformations. The algorithm evaluates all necessary conditions to determine the appropriate transformation, ensuring correctness. □

Algorithm 3 Indeterm-Upside-Down Logic Transformation

Require: Proposition $A \in \mathcal{P}$, context C, current truth value $T(A,C) \in \{\text{True}, \text{False}, \text{Indeterminate}\}$, and evidence E.

Ensure: Transformed truth value $T'(A,C')$.

if $T(A,C) = \text{Indeterminate}$ **then**
  if Evidence E supports A **then**
    Apply U_{IT}: $T'(A,C') \leftarrow \text{True}$.
  else
    Apply U_{IF}: $T'(A,C') \leftarrow \text{False}$.
  end if
else if $T(A,C) = \text{True}$ **then**
  if Evidence E is unreliable **then**
    Apply U_{TI}: $T'(A,C') \leftarrow \text{Indeterminate}$.
  else
    $T'(A,C') \leftarrow T(A,C)$.
  end if
else if $T(A,C) = \text{False}$ **then**
  if Evidence E is unreliable **then**
    Apply U_{FI}: $T'(A,C') \leftarrow \text{Indeterminate}$.
  else
    $T'(A,C') \leftarrow T(A,C)$.
  end if
end if
return $T'(A,C')$.

Theorem 4.25 (Time Complexity)**.** *The time complexity of Algorithm 3 is $O(1)$.*

Proof. The algorithm evaluates the current truth value $T(A,C)$ and evidence E, performing at most two nested conditional checks. As there are no loops or recursive calls, the operations occur in constant time. Hence, the time complexity is $O(1)$. □

Theorem 4.26 (Space Complexity)**.** *The space complexity of Algorithm 3 is $O(1)$.*

Proof. The algorithm requires constant space to store the inputs ($A, C, T(A,C)$, and E) and the output $T'(A,C')$. No additional data structures are instantiated, so the space complexity is $O(1)$. □

Example 4.27 (Medical Diagnosis)**. Input:**

- A: "The patient has disease X."
- C: Initial diagnostic tests are inconclusive.
- $T(A,C) = \text{Indeterminate}$.
- Evidence E: Advanced imaging strongly indicates disease X.

Algorithm Execution: Since $T(A,C) = \text{Indeterminate}$ and E supports A, the algorithm applies U_{IT}:

$$T'(A,C') = \text{True}.$$

Output: The patient's diagnosis transitions to definitive: $T'(A,C') = \text{True}$.

5 Certain Upside-Down Logic

In this section, we explore Certain Upside-Down Logic. This logical framework dynamically adjusts the truth, indeterminacy, and falsity values of propositions based on evidence, redistributing and swapping these components as required.

5.1 Definition of Certain Upside-Down Logic

The definition of Certain Upside-Down Logic is presented below. As a note, in this section, Certain Upside-Down Logic is considered within the framework of Neutrosophic Logic.

Notation 5.1. *In this section, let $\mathcal{P}$ be a set of propositions, and let C be a set of contexts. Let $v : \mathcal{P} \times C \to [0,1]^3$ be a truth valuation function assigning to each proposition-context pair (A, C) a Neutrosophic truth value $v(A, C) = (T, I, F)$, where T, I, and F represent the degrees of **truth**, **indeterminacy**, and **falsity**, respectively, satisfying:*

$$T, I, F \in [0,1], \quad T + I + F = 1.$$

Definition 5.2 (Certain Upside-Down Logic)**.** Certain Upside-Down Logic is an extension of Upside-Down Logic that includes transformations of the truth values based on evidence E. The logic allows for the reallocation of the indeterminacy value I to either the truth value T or the falsity value F, depending on the evidence. Specifically, the transformations are defined as follows:

1. **Weak Evidence Shift (Indeterminacy to Falsity and Swap)**:

   $$v'(A) = (F + I, 0, T),$$

   applied when evidence weakly supports the falsity of A.

2. **Weak Evidence Shift (Indeterminacy to Truth and Swap)**:

   $$v'(A) = (F, 0, T + I),$$

   applied when evidence weakly supports the truth of A.

3. **Strong Evidence Shift (All to Truth)**:

   $$v'(A) = (T + I + F, 0, 0),$$

   applied when evidence strongly supports the truth of A.

4. **Strong Evidence Shift (All to Falsity)**:

   $$v'(A) = (0, 0, T + I + F),$$

   applied when evidence strongly supports the falsity of A.

Remark 5.3 (Transformation Rules of the Certain Upside-Down Logic)**.** The Certain Upside-Down Logic provides a mechanism to update the truth values of propositions based on new evidence, extending the Upside-Down Logic by incorporating the indeterminacy component.

1. **Weak Evidence Shift (Indeterminacy to Falsity and Swap)**:
   - The indeterminacy degree I is shifted to the truth value along with swapping T and F.
2. **Weak Evidence Shift (Indeterminacy to Truth and Swap)**:
   - The indeterminacy degree I is shifted to the falsity value along with swapping T and F.
3. **Strong Evidence Shift (All to Truth)**:
   - All degrees are shifted to the truth value, making $T' = 1$.
4. **Strong Evidence Shift (All to Falsity)**:
   - All degrees are shifted to the falsity value, making $F' = 1$.

5.2 Examples of Real-World Applications of Certain Upside-Down Logic

Certain Upside-Down Logic has broad applicability in scenarios involving dynamic adjustments to truth, indeterminacy, and falsity values based on new evidence. Below are two illustrative examples:

Example 5.4 (Medical Diagnosis)**.** Let A: "The patient has Disease X."

Initial Truth Value: $v(A) = (T, I, F) = (0.3, 0.5, 0.2)$.

Case 1: Weak Evidence Shift (Indeterminacy to Falsity and Swap)

Symptoms weakly suggest the absence of Disease X. Applying the transformation:

$$\begin{aligned} T' &= F + I = 0.2 + 0.5 = 0.7 \\ I' &= 0 \\ F' &= T = 0.3 \end{aligned}$$

Updated Truth Value: $v'(A) = (0.7, 0, 0.3)$.

Case 2: Strong Evidence Shift (All to Falsity)

Tests strongly confirm the absence of Disease X. Applying the transformation:

$$\begin{aligned} T' &= 0 \\ I' &= 0 \\ F' &= T + I + F = 0.3 + 0.5 + 0.2 = 1.0 \end{aligned}$$

Updated Truth Value: $v'(A) = (0, 0, 1.0)$.

Example 5.5 (Weather Prediction)**.** Let B: "It will rain tomorrow."

Initial Truth Value: $v(B) = (T, I, F) = (0.2, 0.4, 0.4)$.

Case 1: Weak Evidence Shift (Indeterminacy to Truth and Swap)

Forecast models weakly support rain. Applying the transformation:

$$\begin{aligned} T' &= F = 0.4 \\ I' &= 0 \\ F' &= T + I = 0.2 + 0.4 = 0.6 \end{aligned}$$

Updated Truth Value: $v'(B) = (0.4, 0, 0.6)$.

Case 2: Strong Evidence Shift (All to Truth)

Satellite data strongly supports rain. Applying the transformation:

$$\begin{aligned} T' &= T + I + F = 0.2 + 0.4 + 0.4 = 1.0 \\ I' &= 0 \\ F' &= 0 \end{aligned}$$

Updated Truth Value: $v'(B) = (1.0, 0, 0)$.

5.3 Basic Theorem of Certain Upside-Down Logic

The Basic Theorem of Certain Upside-Down Logic is described as follows. As a note, in this section, Certain Upside-Down Logic is considered within the framework of Neutrosophic Logic.

Theorem 5.6 (Conservation of Total Degree)**.** *The transformation U_C preserves the total degree of truth, indeterminacy, and falsity:*

$$T + I + F = T' + I' + F' = 1.$$

Proof. As shown in the Algorithm Correctness proof, for each case, the sum $T'+I'+F'$ equals $T+I+F = 1$. □

Theorem 5.7 (Bounds Preservation)**.** *For all transformations, the updated truth values satisfy:*

$$0 \leq T', I', F' \leq 1.$$

Proof. Since $T, I, F \in [0, 1]$ and their sum is 1, and because the transformations are combinations of these values without exceeding their original total, the updated values T', I', and F' remain within $[0, 1]$. □

5.4 Algorithm for Certain Upside-Down Logic

The algorithm updates the Neutrosophic truth value (T, I, F) of a proposition A based on evidence E. The transformation adjusts the degrees of truth, indeterminacy, and falsity according to specific rules, as described below. As a note, in this section, Certain Upside-Down Logic is considered within the framework of Neutrosophic Logic.

Algorithm 4 Update Truth Value in Certain Upside-Down Logic

Require: Proposition A, current truth value $v(A) = (T, I, F)$, evidence E
Ensure: Updated truth value $v'(A) = (T', I', F')$

if E weakly supports the falsity of A **then**
  $T' \leftarrow F + I$
  $I' \leftarrow 0$
  $F' \leftarrow T$
else if E weakly supports the truth of A **then**
  $T' \leftarrow F$
  $I' \leftarrow 0$
  $F' \leftarrow T + I$
else if E strongly supports the truth of A **then**
  $T' \leftarrow T + I + F$
  $I' \leftarrow 0$
  $F' \leftarrow 0$
else if E strongly supports the falsity of A **then**
  $T' \leftarrow 0$
  $I' \leftarrow 0$
  $F' \leftarrow T + I + F$
else
  $T' \leftarrow T$
  $I' \leftarrow I$
  $F' \leftarrow F$
end if
return $v'(A) = (T', I', F')$

Theorem 5.8 (Correctness of Transformation)**.** *For any initial truth value $v(A) = (T, I, F)$ with $T, I, F \geq 0$ and $T + I + F = 1$, the updated truth value $v'(A) = (T', I', F')$ produced by Algorithm 4 satisfies $T', I', F' \geq 0$ and $T' + I' + F' = 1$.*

Proof. We consider each case:

1. **Weak Evidence Shift (Indeterminacy to Falsity and Swap)**:

$$T' = F + I$$
$$I' = 0$$
$$F' = T$$

Since $T + I + F = 1$, we have:

$$T' + I' + F' = (F + I) + 0 + T = T + I + F = 1.$$

All components are non-negative because $T, I, F \geq 0$.

2. **Weak Evidence Shift (Indeterminacy to Truth and Swap)**:

$$T' = F$$
$$I' = 0$$
$$F' = T + I$$

Similarly,

$$T' + I' + F' = F + 0 + (T + I) = T + I + F = 1.$$

3. **Strong Evidence Shift (All to Truth)**:

$$T' = T + I + F = 1$$
$$I' = 0$$
$$F' = 0$$

Thus,

$$T' + I' + F' = 1 + 0 + 0 = 1.$$

4. **Strong Evidence Shift (All to Falsity)**:

$$T' = 0$$
$$I' = 0$$
$$F' = T + I + F = 1$$

Therefore,

$$T' + I' + F' = 0 + 0 + 1 = 1.$$

5. **No Evidence Shift**:

$$T' = T$$
$$I' = I$$
$$F' = F$$

So,

$$T' + I' + F' = T + I + F = 1.$$

In all cases, the updated truth values satisfy $T', I', F' \geq 0$ and $T' + I' + F' = 1$. □

Theorem 5.9 (Time Complexity)**.** *Algorithm 4 runs in constant time, i.e., it has a time complexity of $O(1)$.*

Proof. The algorithm performs a fixed number of operations: conditional checks and assignments. These operations do not depend on the size of the input but are constant. Hence, the time complexity is $O(1)$. □

Theorem 5.10 (Space Complexity)**.** *Algorithm 4 uses constant extra space, i.e., it has a space complexity of $O(1)$.*

Proof. The algorithm requires additional space for variables T', I', and F'. Since the number of variables does not depend on the size of the input, the space complexity is $O(1)$. □

6 Conclusion and Future Work of this Paper

This section presents the conclusion and future directions of this paper.

6.1 Conclusion of this Paper

In this paper, we introduced the following logical frameworks:

- **Upside-Down Logic**: A logical framework that systematically flips truth and falsity by dynamically altering contexts, meanings, or perspectives.
- **Contextual Upside-Down Logic**: An extension of Upside-Down Logic that integrates contextual transformations, enabling the adjustment of logical connectives and truth values based on contextual changes.
- **Indeterm-Upside-Down Logic**: A framework designed to represent real-world phenomena where indeterminacy either increases or decreases, capturing dynamic uncertainty effectively.
- **Certain Upside-Down Logic**: This logic provides a mechanism to update truth values of propositions based on new evidence, extending Upside-Down Logic by incorporating the indeterminacy component for enhanced flexibility.

6.2 Future tasks

Finally, we briefly discuss the future prospects of this research.

6.2.1 Applying Upside-Down Logic to Uncertain Sets and Graphs

As mentioned in the introduction, the study of Upside-Down Logic is still in its early stages. We anticipate further exploration into potential applications of Upside-Down Logic, including considerations such as how to apply Upside-Down Logic to Uncertain Logic. Additionally, we expect advancements in research regarding its applicability to concepts like neutrosophic graphs [**?**, 4, 12, 14, 15, 23, 35, 47, 48, 61, 81], adripartitioned neutrosophic set [76, 77], Neutrosophic Topologies [1, 98], Neutrosophic algebra [53, 95], Heptapartitioned neutrosophic sets [11, 69], Neutrosophic Automata [40, 55–57], Neutrosophic oversets [92, 96, 97], refined neutrosophic logic [90], and Bipolar Neutrosophic Sets [5, 25, 27, 106].

6.2.2 Applying Upside-Down Logic to decision-making

There is also significant potential for applying Upside-Down Logic to decision-making (cf. [3, 46]) and similar fields, which could pave the way for a broader understanding and practical use of this novel logical framework.

6.2.3 Relation to Research on Addressing Indeterminacy

When making decisions or applying theoretical frameworks to real-world scenarios, the ability to effectively handle indeterminacy is often a critical factor. Research aimed at understanding and addressing indeterminacy has been conducted across various fields [20]. Below, we list examples of major research areas:

- **Risk Assessment and Decision-Making:** Approaches to classify and manage uncertainty by analyzing the structural environment of decision-makers. Relevant studies focus on societal risks ([8, 108]).
- **Climate Change Prediction:** Research aims to handle uncertainty in climate change predictions by evaluating variations among climate models and excluding unreliable ones, thereby enhancing projection reliability (cf. [32, 45, 73]).

- **Investment Decision-Making:** The use of real options to evaluate factors such as product demand, production capacity, and investment costs under uncertainty [10, 44, 63], enabling the derivation of optimal investment strategies.

The author is particularly interested in exploring whether combining these research areas with Indeterm-Upside-Down Logic or Neutrosophic Logic could yield intriguing mathematical insights or innovative practical applications.

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Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Disclaimer

This study primarily focuses on theoretical aspects, and its application to practical scenarios has not yet been validated. Future research may involve empirical testing and refinement of the proposed methods. The authors have made every effort to ensure that all references cited in this paper are accurate and appropriately attributed. However, unintentional errors or omissions may occur. The authors bear no legal responsibility for inaccuracies in external sources, and readers are encouraged to verify the information provided in the references independently. Furthermore, the interpretations and opinions expressed in this paper are solely those of the authors and do not necessarily reflect the views of any affiliated institutions.

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